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COMMENTARY
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Key Points:

- The 2021 historic geophysics Nobel was for work on complexity before the disciplines of complexity and geocomplexity were founded
- A selection of key advances in geocomplexity in the last 75 years are reviewed here
- The prize highlights the schism developed following the numerical and nonlinear revolutions. Today, unity can be reclaimed

Supporting Information:

Supporting Information may be found in the online version of this article.

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The 2021 "Complex Systems" Nobel Prize: The Climate, With and Without Geocomplexity

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Abstract One half of 2021s Nobel Physics prize was awarded to statistical physicist Giorgio Parisi and the other—the first in geophysics in 75 years—to climate scientists Syukoro Manabe and Klaus Hasselmann, the former for pioneering General Circulation Models and the latter (primarily) for proposing a statistical model explaining the climate as a slowly varying state driven by random weather noise. However, the Nobel committee recognized the climate laureates' work almost exclusively from the 1960s and 1970s. We update their report with the contributions from nonlinear geophysics and discuss the implications for the unity of geoscience, and for future climate modeling.

Plain Language Summary The first Nobel geophysics prize was awarded to climate scientists Syukoro Manabe and Klaus Hasselmann, for their work primarily in the 1960s and 1970s. Although the prize theme was "complex systems," this was a decade or two before complexity science and geocomplexity were founded. I give a geocomplexity update and argue that it reveals a 40-year-old schism that can now be overcome with important benefits notably for climate projections.

1. Introduction

This year's physics Nobel prize marks history as the first for geophysics since 1947 when Edward Appleton won it for ionospheric research. Half-shared by statistical physicist Georgio Parisi and half by climate scientists Syukoro Manabe and Klauss Hasselmann, the prize was nominally for "complex systems," yet the two halves of the work were so disparate that at least one of the winners (Manabe) reportedly confessed to never having heard of another (Parisi). Ironically, Parisi's important contribution to multifractals was not even mentioned in the committee's 18 page report (Nobel Committee for Physics, 2021) in spite of its significant atmospheric and climate applications (see below). In addition, nonagenarians Manabe and Hasselmann were honored primarily for work in the 1960s and 1970s—before the Nonlinear revolution and before complexity science even existed. In this commentary, I focus on the climate half of the prize giving a succinct update on complexity applied to geoscience: geocomplexity.

Complexity science in general—and geocomplexity in particular—emerged in the wake of the 1980s nonlinear revolution: notably deterministic chaos, fractals, nonlinear waves, self organized criticality and somewhat later, network theory. Complexity physics took shape in the 1990s (see the review, Nicolis & Nicolis, 2012) whereas nonlinear geoscience can be roughly dated from the workshops on Nonlinear VAriability in Geophysics (NVAG 1–4, 1986–1997), the establishment of the Nonlinear Processes division at the European Geophysical Society (now European Geophysical Union, EGU, 1989), the Nonlinear Geophysics focus group at the American Geophysical Union (AGU, 1997) and in 2009, an AGU session with accompanying geocomplexity workshop (Lovejoy et al., 2009). Recently, a group of AGU and EGU geocomplexity scientists collaborated in the establishment of the ongoing Climate Variability Across-Scales working group of PAGES (Past Climate), (Franzke, 2017; Franzke & Yuan, 2020; Laepple et al., 2018; Lovejoy, 2017; Lovejoy et al., 2016), for a review, (Franzke et al., 2020).

2. Geocomplexity and Hasselmann's Picture

We can certainly celebrate the rare geophysics Nobel and its recipients Manabe and Hasselmann, yet the committee's presentation of the pioneers' contributions as almost finished work is problematic. Indeed, their report contains little hint that over the intervening decades geocomplexity has transformed our understanding of the atmosphere and climate system. In this regard, perhaps the most important geocomplexity concept is scaling or scale invariance: a statistical relationship between the dynamics at potentially vastly different spatial and/or temporal scales. Although the concept has been fundamental to turbulence (and hence weather scales) ever since (Richardson, 1926), it has since been systematically applied to longer (e.g., climate) scales (e.g, Eichner et al., 2003; Eghdami et al., 2018; Ellerhoff & Rehfeld, 2021; Franzke et al., 2020; Huybers & Curry, 2006; Laepple & Huybers, 2014; Lovejoy, 2013; Lovejoy, 2015a; Lovejoy & Schertzer, 1986; Nogueira, 2019; Nogueira & Barros, 2014; Pelletier, 1998; Tao & Barros, 2010).

This transformation can be highlighted through a discussion of Hasselmann's climate model. Hasselmann's basic idea is that the weather drives the climate through random internal forcing. In a review and mathematical update (Arnold, 2001), describes Hasselmann's idea as follows: "the slowly responding components of the system (such as the ocean, cryosphere and the biosphere), act as integrators of this random weather input in much the same way as a pollen grain in a liquid integrates the short time impact of the molecules to yields Brownian motion."

While Arnold's reference to a grain of pollen may sound strange, it underscores the generality of the process. In Hasselmann's usage, it applied to the key climate variables, notably the temperature. As explained momentarily, although the details are not quite right, a key point remains: the existence of a fundamental transitional time scale. For the pollen, it separates the high frequency molecular "jitter" dominated by the pollen's inertia, from the low frequency random pollen "walk." In the atmosphere—both regionally and globally and for all fields including temperature, wind, precipitation, humidity—it separates the high frequency weather from the qualitatively different low frequency macroweather and climate regimes.

Arnold's climate analogy with pollen is quantitative: as with pollen, in Hasselmann's model, the system obeys the Langevin equation:

$$\tau \frac{\mathrm{d}T}{\mathrm{d}t} + T = s\gamma(t) \tag{1}$$

where T is the temperature anomaly (or pollen grain velocity), τ is the relaxation time, s a constant (in the climate case, the "climate sensitivity") and $\gamma(t)$ a white noise forcing. The solution of this equation is (by definition) an "Ornstein-Uhlenbeck process;" at high frequencies ($\omega \gg 1/\tau$) its spectrum (E) is that of Brownian motion ($E(\omega) \approx \omega^{-\beta}$ with $\beta = 2$ and ω the frequency) and at low frequencies ($\omega \ll 1/\tau$), it is a white noise ($\beta = 0$, the pollen position—the integral of the velocity—will be a Brownian motion). While Hasselmann derived the Langevin equation on more general grounds (appealing to nonlinear dynamics), the Nobel committee helpfully recalled that the same equation (for the global temperature) is a consequence of the Budyko-Sellers (Budyko, 1969; Sellers, 1969) energy balance model discussed below.

In the application to Earth's energy balance, the white noise $\gamma(t)$ represents the "internal" forcing due to (high frequency) weather and $T(t)$, the "internally forced temperature." If we replace $\gamma(t)$ by an external forcing $F_e(t)$ (in our epoch, mostly anthropogenic but also solar and volcanic), then the solution is the "externally forced temperature." Due to linearity, the combined response to internal and external forcing is the sum of the individual responses. To understand the meaning of the terms in Equation 1, take $F_e(t)$ to be a step function forcing ($F_e(t) = 0$ for $t < 0$, $F_e(t) = F_0$ for $t \geq 0$). At long times, T exponentially (with time constant τ) relaxes to a new equilibrium, the derivative term disappears and $T_{\mathrm{eq}} = sF_0$. At T_{eq}, the extra incoming forcing (F_0) is balanced by an extra outgoing (linearized) black body flux T_{eq}/s. $(\tau/s)\mathrm{d}T/\mathrm{d}t$ is the instantaneous imbalance in the radiative energy fluxes, physically it is the flux of energy stored in the subsurface (in regional models, there is also a term due to the divergence of horizontal heat fluxes).

Hasselmann understood that the scale break at $\omega \approx 1/\tau$ separated the high frequency weather from a fundamentally different lower frequency regime that he identified as the climate. In General Circulation Models (GCMs) (due to deterministic chaos, the "butterfly effect"), at scales of ≈10 days in the atmosphere (and months to ≈ a year in the oceans), it separates high frequency deterministic from low frequency stochastic behavior. However since then, our understanding of the nature of both high and low frequency regimes has evolved so that Hasselman's transition scale is close to the lifetimes of planetary scale structures marking the weather—macroweather transition scale (Lovejoy, 2013) which is in general different from the relaxation time τ in Equation 1.

To understand this further, consider Figure 1 that Hasselmann used to empirically justify his model (from Frankignoul and Hasselmann (1977), helpfully reproduced in Nobel Committee for Physics (2021)). It shows the spectrum of a commonly used climate surrogate: a single Sea Surface Temperature (SST, note the large uncertainty). Also shown is a modern update using many more (452) ocean SST series with each 10 times longer

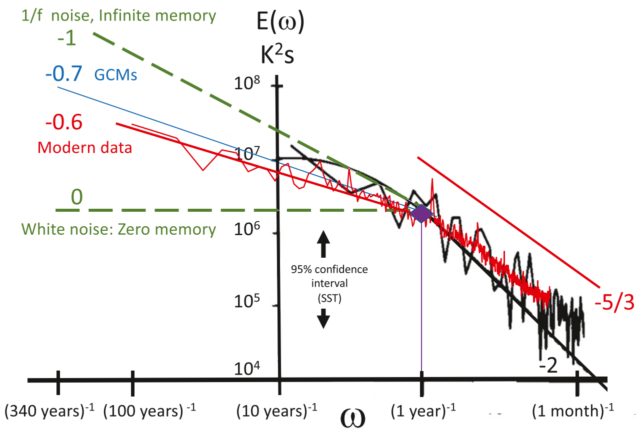

Figure 1. The black spectrum (including the reference line with −2 slope) is from a single North Atlantic Sea Surface Temperature (SST) series over the frequency range (1 month)$^{-1}$ to (10 years)$^{-1}$ (from Frankignoul and Hasselmann (1977) reproduced by the Nobel Committee for Physics (2021)) with the original 95% confidence interval indicated). The figure has been updated with superposed spectrum from 452 SST series from 1911 to 2010 (red, from Lovejoy & Schertzer, (2012)), the series are at 5° × 5° and monthly resolutions and are globally representative). The red reference line on the left (low frequencies) indicates scaling behaviors $E(\omega) \approx \omega^{-\beta}$ with $\beta = 0.6$. Also shown (blue line to (340 years)$^{-1}$) is the scaling behavior ($\beta = 0.7$) inferred from analyses of globally averaged temperatures from control runs from 11 different General Circulation Models (GCMs), (Lovejoy, 2019). These were the GCMs that participated in the Climate Model Intercomparison Project 5 and that provided control runs at least 400 years long. (A control run is the result of running the model with fixed external boundary conditions, e.g., no anthropogenic forcing). Finally, several theoretical scaling behaviors (straight lines on this log-log plot) are shown: at high frequencies Brownian motion $\beta = 2$ as well as the turbulent $\beta = 5/3$ spectrum (red reference line at the right, the data are closer to $\beta = 1.8$). Then, at low frequencies, the dashed green lines with $\beta = 0$ and $\beta = 1$ indicate white noise and 1/*f* noise respectively. Gaussian processes with $0 < \beta \leq 1$ have long memories varying between the extremes of 0 and infinity respectively.

(centennial scales). At the high frequencies we see that the absolute slope is closer to $\beta \approx 1.8$ than to the Brownian motion value $\beta = 2$. While this difference may seem small, it has been amply confirmed by numerous analyses (e.g., Monetti et al. (2003), reviewed in Lovejoy and Schertzer (2013)) and it corresponds to a different understanding of the ocean. Whereas Hasselmann's $\beta = 2$ corresponds to the integral of an (uncorrelated, unstructured) white noise, the value $\beta \approx 1.8$ is close to Kolmogorov's turbulent value (5/3) and is a consequence of ocean structures that are turbulent and spatially scaling (and strongly interacting) up to planetary scales.

Turning to the lower frequencies, rather than a flat ($\beta = 0$, white noise) spectrum, we find $\beta \approx 0.6$ (SST data, see (Eichner et al., 2003; Koscielny-Bunde et al., 1998; Monetti et al., 2003)). Also shown is the $\beta \approx 0.7$ spectrum inferred from analyses of 11 multicentennial GCM outputs with fixed external forcing ("control runs," (Lovejoy, 2019), see also (Rybski et al., 2008)). Once again, while the data and GCMs may appear to be close to Hasselmann's model, the qualitative differences are vast. While Gaussian processes with $\beta = 0$ are completely unpredictable, processes with $0 < \beta \leq 1$ have long range memories that (increasing with β) may be so large that—with enough past data—they may be *infinitely* predictable (the $\beta = 1$ limit). Such processes are aptly called "long memory processes" (or "Hurst phenomena") that were studied notably by (Hurst, 1951) (the Nile river), and by (Mandelbrot & Van Ness, 1968; Mandelbrot & Wallis, 1968) (the "Joseph effect") and has received increasing attention in nonlinear geoscience (e.g, Bunde et al., 2005; Lovejoy, 2015b; Lovejoy and Schertzer, 1986; Rypdal et al., 2013).

Due to the huge memory, even in practice, monthly and seasonal forecasts become "pastvalue"—not classical initial value—problems meaning that the optimum forecast is obtained by using as much past data as possible. Surprisingly, using additional data from other locations (e.g., "teleconnections") does not improve the forecast: the other locations have no Granger causality: in long enough series, the information from these—even strong spatially correlated locations including El Nino regions—have "already been used" and do not add any extra skill (Del Rio Amador & Lovejoy, 2021a). The Stochastic Seasonal and Interannual Prediction System (StocSIPS, (Del Rio Amador & Lovejoy, 2021b), and (Del Rio Amador & Lovejoy, 2019; Del Rio Amador & Lovejoy, 2021b)) show how this huge memory can be exploited to make long range temperature forecasts (for climate projections, see below).

The source of the temporal scaling in both low and high frequency regimes is not mysterious. Both ultimately have their origins in the wide range spatial scaling of the equations governing the atmosphere and ocean ((Schertzer et al., 2012) and boundary conditions, see the review (Lovejoy & Schertzer, 2013)). The transition time scale in Figure 1 at around 1 year, is simply the typical lifetime of planetary scale ocean structures and the analogous atmospheric transition (typically at about 10 days) corresponds to the much shorter lifetimes of atmospheric structures (on Mars, the analogous transition is at ≈2 days (Chen et al., 2016)). For Manabe's heritage, this disagreement between the data and Hasselmann's model is fortunate: GCMs inherit the scaling from the governing equations so that their statistics (including the weather regime's multifractal intermittencies) quantitatively agree with the data.

3. The Origin of Macroweather Scaling

What is new and exciting, is that the precise origin of this long range memory can now be better understood thanks to an updated derivation of Hasselmann's equation. As recalled in (Nobel Committee for Physics, 2021), the usual approach starts with the energy balance of the earth with outer space. As mentioned, the forcing term in Equation 1 (the right hand side) includes not only white noise (from "internal variability"), but also the imbalance in energy flux due to anthropogenic and other external causes. Some of this energy is stored in the earth's subsurface and can emerge decades later (the far left term), and some raises the temperature thus increasing the outgoing black body radiation to outer space (the second term on the left). Equation 1 is the classical Energy Balance Equation (EBE).

In the original EBE derivation, the (vertical) radiative imbalance at any point on the Earth was simply redirected toward the poles an approximation leading to the (order one) derivative term in the EBE ($\tau dT/dt$), for the energy storage with its fast (exponential) relaxation following a perturbation. However, it was recently found that if the (correct) radiative—conductive surface boundary conditions are used, that the result is a (fractional) Half-order Energy Balance Equation (HEBE) where the derivative term in Equation 1 is replaced by $\tau^{1/2}d^{1/2}T/dt^{1/2}$ (Lovejoy, 2021a, 2021b). Fractional derivatives are convolutions with power laws—or equivalently, in the frequency domain—they are power law filters. In the HEBE, they imply a spectrum $\beta = 1$, that is, a (very!) long memory (power law) relaxation process: power law energy storage. Minor HEBE generalizations yield the Fractional EBE (the FEBE with derivative term $\tau^h d^h T/dt h$ (Lovejoy et al., 2021)) that is compatible with the observed β values. Empirically, from the response to deterministic anthropogenic forcing, $h = 0.38 \pm 0.03$, (Procyk et al., 2022), and from the response to stochastic internal forcing, $h = 0.42 \pm 0.02$, (Del Rio Amador & Lovejoy, 2019), both are close to the HEBE value $h = 1/2$.

4. Intermittency, Multifractals and Parisi's Contribution

We mentioned that the FEBE (i.e., the fractionally updated Equation 1) is driven by both external (including anthropogenic) deterministic forcing as well as (internal) Gaussian white noise. While the latter hypothesis seems natural—and it is indeed fairly realistic for this macroweather regime—it turns out that this Gaussian behavior is actually quite exceptional, that geostatistics are on the contrary generally highly non-Gaussian, they are intermittent: this is where Parisi and multifractals come in.

Intermittency can be defined as "the sudden transition from quiescence to chaos" although these transitions are often hidden from view, they can be revealed by the simple expedient of a "spike plot" (Lovejoy, 2018) (Figure 2). From a time series $T(t)$, spike plots are easy to construct. If time is measured in integer units of the resolution, first take the absolute first difference: $\Delta T(t) = |T(t) - T(t-1)|$ with mean (indicated with an overbar): $\overline{\Delta T}$ the spike plot is the normalized series: $\Delta T(t)/\overline{\Delta T}$. Figure 2 (top row) shows the result in time for three series in different scaling regimes, the bottom row shows the analogous spatial trajectory and transect results. With the single exception of macroweather in time (top row, middle), the plots reveal transitions that are so strong that even on the short series in the figure—were the processes Gaussian—their probabilities of occurrence would often be lower than 10^{-12}. If the series are scaling, then their statistics only depend on the total range of scales covered by the plot (here, a factor of 360).

Although the spikes may seem extreme, they are easily understood and modeled with the help of multifractals whose huge spikes are simply singularities of random orders whose probability distribution has a scale invariant exponent. This probability exponent is a "codimension function" and it quantifies the probabilities over scale ranges that may span many orders of magnitude (Schertzer & Lovejoy, 1987). The temporal macroweather exception corresponds to quasi-Gaussian internal variability, compatible with the internal forcing in Equation 1. In the left column (weather regime), the figure visually displays the strong turbulent (weather) intermittency that culminated in the 1980s with the discovery of multifractals and an understanding of their generic process, cascades. In multifractals, the variability builds up scale after scale, so that while numerical weather models do capture the cascades and multifractality with some accuracy (Stolle et al., 2009, 2012), although they lack a wide enough range of scales to accurately reproduce the extremes. Fortunately, for macroweather in time (top middle), the intermittency is much smaller so that Gaussian approximations may be adequate for many purposes.

The understanding of these huge multifractal spikes—their origin in scaling dynamics as well as their implications—was the work of decades, and it included an important early contribution by Nobel laureate Parisi who not only coined the term "multifractal" but also pointed out an important and elegant link between multifractal statistical moments and probability distributions (Parisi & Frisch, 1985).

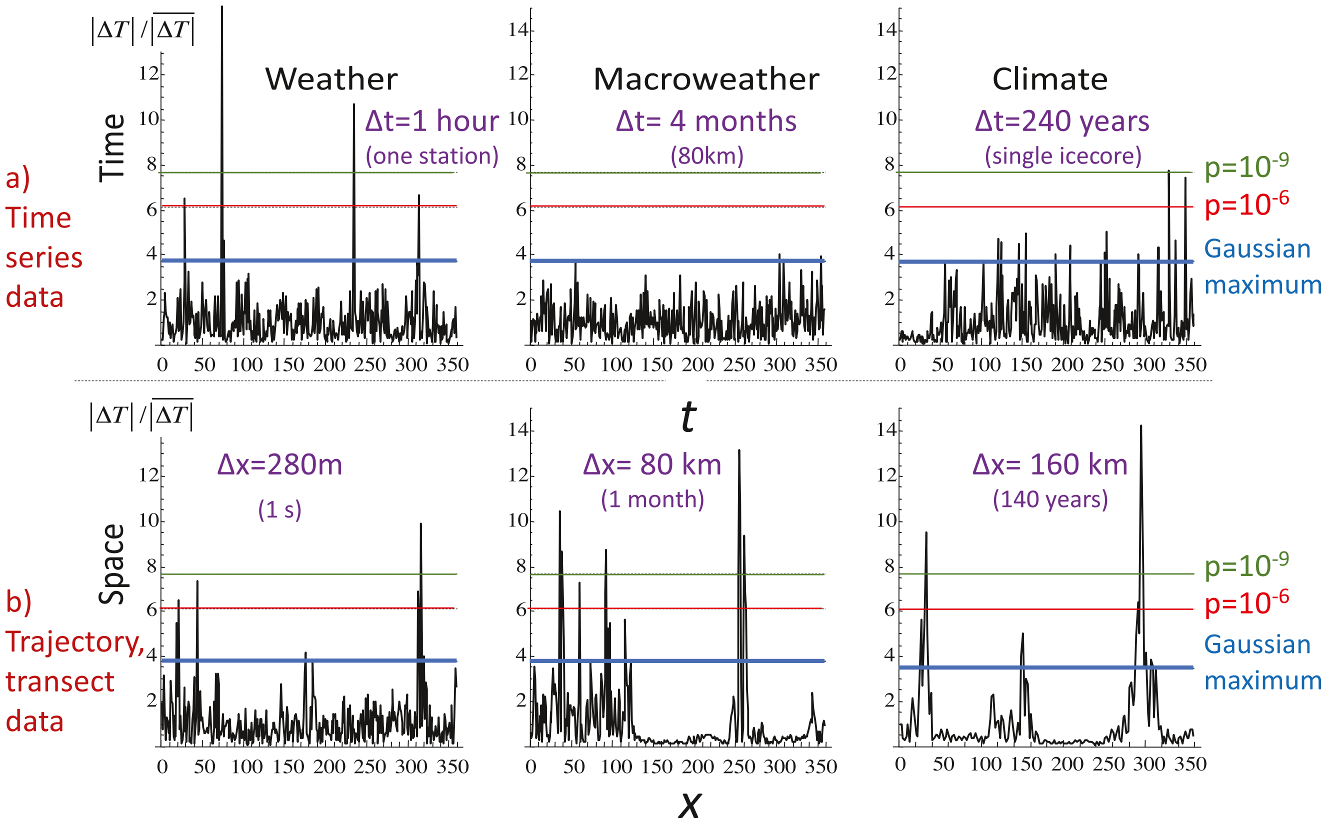

Figure 2. Temperature spike plots for weather (time scales ≈<10 days, lifetimes of planetary scale structures), macroweather (lifetimes of many planetary scale structures, fluctuations converging at longer and longer time scales), climate (lifetimes of many planetary scale structures but with strong low frequencies such that fluctuations no longer converge, they grow with time scale up until Milankovitch—iceage—scales). The top (time series) row is in nondimensional time that is, with t in units of the resolution Δt indicated. The second (spatial) row is in nondimensional space (x, in units of the corresponding spatial resolution Δx). As described in the text, the vertical scale shows the corresponding nondimensional "spikes" that are simply the absolute first difference (i.e., increment) of the original series (time) or trajectory, transect (space) divided by its average. The thick horizontal blue line indicates the expected maximum for a Gaussian process with the same number of points (360 for each with the exception of the lower right which had only 180 points), the red, green lines are the corresponding Gaussian probability levels $p = 10^{-6}$, $p = 10^{-9}$ respectively, a ratio of 14 corresponds to a Gaussian $p \approx 10^{-77}$. Data used in the figure: Montreal at 1 hr resolution (upper left); Montreal at 4 months resolution (upper middle); paleotemperatures from Greenland ice cores at 240 years resolution (upper right); an aircraft trajectory at 280 m resolution (lower left); and from reanalysis at monthly resolution temperatures at 1° spatial resolution (45°N, lower middle) and at 140 years resolution in time, 2° in space (at 45°N, lower right). Adapted from (Lovejoy, 2019).

5. Conclusions

This brief update underscores the historically poor connections between atmospheric science and nonlinear geophysics: over the decades, the respective scientific communities have only weakly—and intermittently—interacted. The problem is hardly new. Since at least (Richardson, 1922), atmospheric science has been under tension between its deterministic and statistical strands: idealized mechanisms and numerical models versus stochastic turbulence approaches. In a recent non-technical book (Lovejoy, 2019), I have argued that the conjunction of numerical and nonlinear revolutions in the 1970s provoked a schism with the two strands developing largely in parallel.

Today—four decades later—these strands can be re-united. This is possible on the one hand because by persistently modeling smaller and smaller structures—"chasing the details"—the numerical models have become big enough—and accurate enough—to display the statistical behavior predicted by nonlinear geophysics—including their scaling and intermittency. On the other hand, the nonlinear (statistical) strand has spawned realistic stochastic models that statistically account for the collective dynamics of huge numbers of interacting structures and processes. The two approaches are thus rapidly converging as it becomes increasingly clear that just as with statistical mechanics and thermodynamics, both levels of understanding can co-exist without contradiction, and both can be profitably exploited, for example, in hybrid models (below).

To see how this convergence may inform future climate models, recall that at present, climate models are essentially low resolution weather models that aim to simulate as many detailed structures and processes as possible. For years, it has been argued (e.g, Shukla et al., 2009; Slingo et al., 2021) that by 2030, kilometric scale, "seamless" weather—climate models will lead to "a quantum leap" improvement in multidecadal projections (Shukla et al., 2009). Yet, such models would simulate structures that live for only 15 min! For the multidecadal projections required to discern the consequences of anthropogenic forcing, these structures contribute high frequency noise that must be averaged out—by factors approaching 1 million (i.e., 10 years/15 min)!

Yet improvement is urgent. For decades, the uncertainties in climate projections have been very large—for example, the famous range of 1.5 C–4.5 C/CO_2 doubling (the "climate sensitivity" for a doubling in CO_2) has hardly changed since the first model estimates in 1979 (Climate Research Board, 1979). The 3 C range is already so large that there is a "disconnect" between climate policies (mitigation) and consequences (temperature increases), giving policy makers too much "wiggle room." Yet for a long time, there was hope that a new generation of climate models would reduce the uncertainties to a narrower, more politically constraining range. Unfortunately, this hope is now shattered. The latest (6th) Assessment Report of the Intergovernmental Panel on Climate Change (IPCC, AR6, 2021): finds that since the previous generation AR5 models (2013) that projection uncertainties have substantially *grown*.

To understand this, recall that IPCC projections are made using dozens of different GCMs produced by different teams all around the world, this is the "multi-model ensemble" (MME). The median MME projection is the most likely and the spread of the different projections about the median defines the "structural uncertainty," conventionally taken as the bounds containing 90% of the individual projections. Whereas the IPCC's AR5 (2013) projected a rise of 1.9 C–4.5 C/CO_2 doubling, this wide 2.6 C range was substantially *increased* in the AR6 (2021) to 3.5 C (2 C–5.5 C/CO_2 doubling, all with 90% probabilities). It seems that while each team had diligently improved its own GCM by implementing higher spatial and temporal model resolutions and modeling more physical processes, the overall result was to drive their projections further away from those of the other teams who improved their models in slightly different ways, making different assumptions, parametrizations and algorithms (Meehl et al., 2020). The tendency of models to become more and more different from each other implies a growing spread in the MME quantified by a growing MME uncertainty (Lovejoy, 2022).

By exploiting the stochastic side of atmospheric science, it may now be possible to jettison the irrelevant details and to construct stochastic, scaling models directly in the macroweather regime using the parameters relevant for the statistics. For example, the FEBE mentioned above based on scale and energy conservation symmetries can already make state-of-the-art stochastic long range forecasts as well as climate projections, with the latter's "parametric uncertainty" about half the MME's structural uncertainty (Procyk et al., 2022). Although only in their infancy, these FEBE based models already directly simulate the statistics of infinite ensembles while replacing supercomputers with laptops (Lovejoy, 2022). Better still: GCMs and stochastic macroweather models may well be compatible with each other in the same way that statistical mechanics and thermodynamics are mutually compatible. This leads to the possibility of using the two different levels of modeling in "hybrid" mode to obtain climate projections that are better still (Lovejoy & Hébert, 2018).

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

This paper used plots adapted from the various papers that have been clearly referenced. The provenance of the data is clearly indicated in these publications. For Figure 1, these are: ((Frankignoul & Hasselmann, 1977) reproduced by the Nobel Committee for Physics (2021) with the original 95% confidence interval indicated). The spectrum from 452 SST series from 1911 to 2010 (red, from Lovejoy & Schertzer, 2012). The black line inferred from analyses of globally averaged temperatures from control runs from 11 different GCMs, (Lovejoy, 2019). For Figure 2, these are described in detail in Lovejoy (2019). No other data was used.

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